WJEC GCE AS and A LEVEL in FURTHER MATHEMATICS

For teaching from 2017
For AS award from 2018
For A level award from 2019

This specification meets the Approval Criteria for GCE AS and A Level Further Mathematics and the GCE AS and A Level Qualification Principles which set out the requirements for all new or revised GCE specifications developed to be taught in Wales from September 2017.

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GCE AS and A LEVEL FURTHER MATHEMATICS (Wales)
SUMMARY OF ASSESSMENT

This specification is divided into a total of 5 units, 3 AS units and 2 A2 units. Weightings noted below are expressed in terms of the full A level qualification.

All AS units and A2 Unit 4 are compulsory.

AS (3 units)

AS Unit 1: Further Pure Mathematics A Written examination: 1 hour 30 minutes $13\frac{1}{3}$% of qualification	70 marks
The paper will comprise a number of short and longer, both structured and unstructured questions, which may be set on any part of the subject content of the unit. A number of questions will assess learners' understanding of more than one topic from the subject content. A calculator will be allowed in this examination.	
AS Unit 2: Further Statistics A Written examination: 1 hour 30 minutes $13\frac{1}{3}$% of qualification	70 marks
The paper will comprise a number of short and longer, both structured and unstructured questions, which may be set on any part of the subject content of the unit. A number of questions will assess learners' understanding of more than one topic from the subject content. A calculator will be allowed in this examination.	
AS Unit 3: Further Mechanics A Written examination: 1 hour 30 minutes $13\frac{1}{3}$% of qualification	70 marks
The paper will comprise a number of short and longer, both structured and unstructured questions, which may be set on any part of the subject content of the unit. A number of questions will assess learners' understanding of more than one topic from the subject content. A calculator will be allowed in this examination.	

A Level (the above plus a further 2 units)

Candidates must take Unit 4 and either Unit 5 or Unit 6.

A2 Unit 4: Further Pure Mathematics B Written examination: 2 hours 30 minutes 35% of qualification	120 marks
This unit is **compulsory**. The paper will comprise a number of short and longer, both structured and unstructured questions, which may be set on any part of the subject content of the unit. A number of questions will assess learners' understanding of more than one topic from the subject content. A calculator will be allowed in this examination	
A2 Unit 5: Further Statistics B Written examination: 1 hour 45 minutes 25% of qualification	80 marks
Learners will sit **either** Unit 5 **or** Unit 6. The paper will comprise a number of short and longer, both structured and unstructured questions, which may be set on any part of the subject content of the unit. A number of questions will assess learners' understanding of more than one topic from the subject content. A calculator will be allowed in this examination	
A2 Unit 6: Further Mechanics B Written examination: 1 hour 45 minutes 25% of qualification	80 marks
Learners will sit **either** Unit 5 **or** Unit 6. The paper will comprise a number of short and longer, both structured and unstructured questions, which may be set on any part of the subject content of the unit. A number of questions will assess learners' understanding of more than one topic from the subject content. A calculator will be allowed in this examination	

This is a unitised specification which allows for an element of staged assessment. Assessment opportunities will be available in the summer assessment period each year, until the end of the life of the specification.

Unit 1, Unit 2 and Unit 3 will be available in 2018 (and each year thereafter) and the AS qualification will be awarded for the first time in summer 2018.

Unit 4, Unit 5 and Unit 6 will be available in 2019 (and each year thereafter) and the A level qualification will be awarded for the first time in summer 2019.

Qualification Accreditation Numbers
GCE AS: XXXXXXX
GCE A level: XXXXXXXX

GCE AS AND A LEVEL FURTHER MATHEMATICS

1 INTRODUCTION

1.1 Aims and objectives

This WJEC GCE AS and A Level in Further Mathematics provides a broad, coherent, satisfying and worthwhile course of study. It encourages learners to develop confidence in, and a positive attitude towards, mathematics and to recognise its importance in their own lives and to society. The specification has been designed to respond to the proposals set out in the report of the ALCAB panel on mathematics and further mathematics.

The WJEC GCE AS and A level in Further Mathematics encourages learners to:

- develop their understanding of mathematics and mathematical processes in a way that promotes confidence and fosters enjoyment;
- develop abilities to reason logically and recognise incorrect reasoning, to generalise and to construct mathematical proofs;
- extend their range of mathematical skills and techniques and use them in more difficult, unstructured problems;
- develop an understanding of coherence and progression in mathematics and of how different areas of mathematics can be connected;
- recognise how a situation may be represented mathematically and understand the relationship between 'real world' problems and standard and other mathematical models and how these can be refined and improved;
- use mathematics as an effective means of communication;
- read and comprehend mathematical arguments and articles concerning applications of mathematics;
- acquire the skills needed to use technology such as calculators and computers effectively, recognise when such use may be inappropriate and be aware of limitations;
- develop an awareness of the relevance of mathematics to other fields of study, to the world of work and to society in general;
- take increasing responsibility for their own learning and the evaluation of their own mathematical development.

1.2 Prior learning and progression

Any requirements set for entry to a course following this specification are at the discretion of centres. It is reasonable to assume that many learners will have achieved qualifications equivalent to Level 2 at KS4. Skills in Numeracy/Mathematics, Literacy/English and Information and Communication Technology will provide a good basis for progression to this Level 3 qualification. Candidates may be expected to have obtained (or to be obtaining concurrently) an Advanced GCE in Mathematics.

This specification builds on the knowledge, understanding and skills established at GCSE.

This specification provides a suitable foundation for the study of mathematics or a related area through a range of higher education courses, progression to the next level of vocational qualifications or employment. In addition, the specification provides a coherent, satisfying and worthwhile course of study for learners who do not progress to further study in this subject.

This specification is not age specific and, as such, provides opportunities for learners to extend their life-long learning.

1.3 Equality and fair access

This specification may be followed by any learner, irrespective of gender, ethnic, religious or cultural background. It has been designed to avoid, where possible, features that could, without justification, make it more difficult for a learner to achieve because they have a particular protected characteristic.

The protected characteristics under the Equality Act 2010 are age, disability, gender reassignment, pregnancy and maternity, race, religion or belief, sex and sexual orientation.

The specification has been discussed with groups who represent the interests of a diverse range of learners, and the specification will be kept under review.

Reasonable adjustments are made for certain learners in order to enable them to access the assessments (e.g. candidates are allowed access to a Sign Language Interpreter, using British Sign Language). Information on reasonable adjustments is found in the following document from the Joint Council for Qualifications (JCQ): *Access Arrangements and Reasonable Adjustments: General and Vocational Qualifications.*

This document is available on the JCQ website (www.jcq.org.uk). As a consequence of provision for reasonable adjustments, very few learners will have a complete barrier to any part of the assessment.

1.4 Welsh Baccalaureate

In following this specification, learners should be given opportunities, where appropriate, to develop the skills that are being assessed through the Skills Challenge Certificate within the Welsh Baccalaureate:

- Literacy
- Numeracy
- Digital Literacy
- Critical Thinking and Problem Solving
- Planning and Organisation
- Creativity and Innovation
- Personal Effectiveness.

1.5 Welsh perspective

In following this specification, learners should be given opportunities, where appropriate, to consider a Welsh perspective if the opportunity arises naturally from the subject matter and if its inclusion would enrich learners' understanding of the world around them as citizens of Wales as well as the UK, Europe and the world.

2 SUBJECT CONTENT

Mathematics is, inherently, a sequential subject. There is a progression of material through all levels at which the subject is studied. The specification content therefore builds on the skills, knowledge and understanding set out in the whole GCSE subject content for Mathematics and Mathematics-Numeracy for first teaching from 2015. It also builds upon the skills, knowledge and understanding in AS and A level Mathematics.

Overarching themes

This GCE AS and A Level specification in Further Mathematics requires learners to demonstrate the following overarching knowledge and skills. These must be applied, along with associated mathematical thinking and understanding, across the whole of the detailed content set out below. The knowledge and skills required for AS Further Mathematics are shown in bold text. The text in standard type applies to A2 only.

Mathematical argument, language and proof

GCE AS and A Level Further Mathematics specifications must use the mathematical notation set out in Appendix A and must require learners to recall the mathematical formulae and identities set out in Appendix B.

Knowledge/Skill
Construct and present mathematical arguments through appropriate use of diagrams; sketching graphs; logical deduction; precise statements involving correct use of symbols and connecting language, including: constant, coefficient, expression, equation, function, identity, index, term, variable
Understand and use mathematical language and syntax as set out in the content
Understand and use language and symbols associated with set theory, as set out in the content
Understand and use the definition of a function; domain and range of functions
Comprehend and critique mathematical arguments, proofs and justifications of methods and formulae, including those relating to applications of mathematics

Mathematical problem solving

Knowledge/Skill
Recognise the underlying mathematical structure in a situation and simplify and abstract appropriately to enable problems to be solved
Construct extended arguments to solve problems presented in an unstructured form, including problems in context
Interpret and communicate solutions in the context of the original problem
Understand the concept of a mathematical problem solving cycle, including specifying the problem, collecting information, processing and representing information and interpreting results, which may identify the need to repeat the cycle
Understand, interpret and extract information from diagrams and construct mathematical diagrams to solve problems

Mathematical modelling

Knowledge/Skill
Translate a situation in context into a mathematical model, making simplifying assumptions
Use a mathematical model with suitable inputs to engage with and explore situations (for a given model or a model constructed or selected by the learner)
Interpret the outputs of a mathematical model in the context of the original situation (for a given model or a model constructed or selected by the learner)
Understand that a mathematical model can be refined by considering its outputs and simplifying assumptions; evaluate whether the model is appropriate
Understand and use modelling assumptions

Use of data in statistics

This specification requires learners, during the course of their study, to:

- develop skills relevant to exploring and analysing large data sets (these data must be real and sufficiently rich to enable the concepts and skills of data presentation and interpretation in the specification to be explored);
- use technology such as spreadsheets or specialist statistical packages to explore data sets;
- interpret real data presented in summary or graphical form;
- use data to investigate questions arising in real contexts.

Learners should be able to demonstrate the ability to explore large data sets, and associated contexts, during their course of study to enable them to perform tasks, and understand ways in which technology can help explore the data. Learners should be able to demonstrate the ability to analyse a subset or features of the data using a calculator with standard statistical functions.

2.1 AS UNIT 1

Unit 1: Further Pure Mathematics A
Written examination: 1 hour 30 minutes
$13\frac{1}{3}$% of A level qualification ($33\frac{1}{3}$% of AS qualification)
70 marks

The subject content is set out on the following pages. There is no hierarchy implied by the order in which the content is presented, nor should the length of the various sections be taken to imply any view of their relative importance.

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in AS Mathematics.
Where specific content requires knowledge of concepts or results from A2 Mathematics, this will be made explicit in the Guidance section of the content.

Topics	Guidance
2.1.1 Proof	
Construct proofs using mathematical induction. Contexts include sums of series, powers of matrices and divisibility.	Including application to the proof of the binomial theorem for a positive integral power. eg. the proof of the divisibility of $5^{2n} - 1$ by 24. *Knowledge of the* Σ *notation is assumed.*
2.1.2 Complex Numbers	
Solve any quadratic equation with real coefficients. Solve cubic or quartic equations with real coefficients (given sufficient information to deduce at least one root for cubics or at least one complex root or quadratic factor for quartics).	
Add, subtract, multiply and divide complex numbers in the form $x + \mathrm{i}y$, with x and y real. Understand and use the terms 'real part' and 'imaginary part'.	

Topics	Guidance
Understand and use the complex conjugate. Know that non-real roots of polynomial equations with real coefficients occur in conjugate pairs	The complex conjugate of z will be denoted by $\overline{z}$.
Equate the real and imaginary parts of a complex number.	Including the solution of equations such as $z + 2\overline{z} = \dfrac{1+2\mathrm{i}}{1-\mathrm{i}}$.
Use and interpret Argand diagrams	Includes representing complex numbers by points in an Argand diagram.
Understand and use the Cartesian (algebraic) and modulus-argument (trigonometric) forms of a complex number. Convert between the Cartesian form and modulus-argument form of a complex number.	$z = x + \mathrm{i}y$ and $z = r(\cos\theta + \mathrm{i}\sin\theta)$ where $\theta = \arg(z)$ may be taken to be in either $[0, 2\pi)$ or $(-\pi, \pi]$ or $[0, 360^{\mathrm{o}})$ or $(-180^{\mathrm{o}}, 180^{\mathrm{o}}]$. *Knowledge of radians is assumed.*
Multiply and divide complex numbers in modulus-argument form.	*Knowledge of radians and compound angle formulae is assumed.*
Construct and interpret simple loci in an Argand diagram, such as $\lvert z-a\rvert > r$ and $\arg(z-a) = \theta$.	For example, $\lvert z-1\rvert = 2\lvert z+\mathrm{i}\rvert$. *Knowledge of radians is assumed.*
Simple cases of transformations of lines and curves defined by $w = f(z)$.	For example, the image of the line $x + y = 1$ under the transformation defined by $w = z^2$.

Topics	Guidance
2.1.3 Matrices	
Add, subtract and multiply conformable matrices. Multiply a matrix by a scalar.	
Understand and use zero and identity matrices. Understand and use the transpose of a 2 x 2 matrix.	
Use matrices to represent • linear transformations in 2-D, • successive transformations, • single transformations in 3-D (3-D transformations confined to reflection in one of x = 0, y = 0, z = 0 or rotation about one of the coordinate axes).	Transformations to include translation, rotation and reflection. Knowledge that the transformation represented by **AB** is the transformation represented by **B** followed by the transformation represented by **A**. *Knowledge of 3-D vectors is assumed.*
Find invariant points and lines for a linear transformation.	
Calculate determinants of 2 x 2 matrices.	Use and understand the notation $\lvert\mathbf{M}\rvert$ or det **M** or $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ or Δ.
Understand and use singular and non-singular matrices. Understand and use properties of inverse matrices. Calculate and use the inverse of non-singular 2 x 2 matrices.	
2.1.4 Further Algebra and Functions	
Understand and use the relationship between roots and coefficients of polynomial equations up to quartic equations.	
Form a polynomial equation whose roots are a linear transformation of the roots of a given polynomial equation (of at least cubic degree).	

Topics	Guidance
Understand and use formulae for the sums of integers, squares and cubes and use these to sum other series. Understand and use the method of differences for summation of series, including the use of partial fractions.	Summation of a finite series. Use of formulae for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$. Including mathematical induction (see section on Proof) and difference methods. Summation of series such as $\sum_{r=1}^{n} \frac{1}{r(r+1)}$ and $\sum_{r=1}^{n} (2r+1)^3$. *Knowledge of the Σ notation and partial fractions is assumed.*
2.1.5 Further Vectors	
Understand and use the vector and Cartesian forms of an equation of a straight line in 3-D.	$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b}$ and $\frac{x-a_1}{b_1} = \frac{x-a_2}{b_2} = \frac{x-a_3}{b_3}$ *Knowledge of 3-D vectors is assumed.*
Understand and use the vector and Cartesian forms of the equation of a plane.	
Calculate the scalar product and use it to express the equation of a plane, and to calculate the angle between two lines, the angle between two planes and the angle between a line and a plane.	$\mathbf{a}.\mathbf{b} = \|\mathbf{a}\|\|\mathbf{b}\|\cos\theta$ The form $\mathbf{r}.\mathbf{n} = k$ for a plane.
Use the scalar product to check whether vectors are perpendicular.	
Find the intersection of a line and a plane.	
Calculate the perpendicular distance between two lines, from a point to a line and a point to a plane.	

2.2 AS UNIT 2

Unit 2: Further Statistics A
Written examination: 1 hour 30 minutes
$13\frac{1}{3}$% of A level qualification ($33\frac{1}{3}$% of AS qualification)
70 marks

The subject content is set out on the following pages. There is no hierarchy implied by the order in which the content is presented, nor should the length of the various sections be taken to imply any view of their relative importance.

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in AS Mathematics.
Where specific content requires knowledge of concepts or results from A2 Mathematics, this will be made explicit in the Guidance section of the content.

Topics	Guidance
2.2.1 Random Variables and the Poisson Process	
Understand and use the mean and variance of linear combinations of independent random variables. ie. use of results: $E(aX + b) = aE(X) + b$ $Var(aX + b) = a^2Var(X)$ $E(aX + bY) = aE(X) + bE(Y)$ For independent X and Y, use the results: $E(XY) = E(X)\,E(Y)$ $Var(aX + bY) = a^2Var(X) + b^2Var(Y)$	For discrete and continuous random variables.
Probability: Discrete probability distributions. Find the mean and variance of simple discrete probability distributions.	Use of $E(X) = \mu = \sum xP(X = x)$ and $Var(X) = \sigma^2 = \sum x^2P(X = x) - \mu^2$

Topics	**Guidance**
Probability: Continuous probability distributions. Understand and use probability density and cumulative distribution functions and their relationships. Find and use the median, quartiles and percentiles. Find and use the mean, variance and standard deviation. Understand and use the expected value of a function of a continuous random variable.	Use of the results $f(x) = F'(x)$ and $F(x) = \int_{-\infty}^{x} f(t)\,\mathrm{d}t$. E[$g(X)$] = $\int g(x)\,f(x)\,\mathrm{d}x$ Simple functions only, e.g. $\frac{1}{X^2}$ and $\sqrt{X}$.
Statistical distributions: Poisson and exponential distributions. Find and use the mean and variance of a Poisson distribution and an exponential distribution. Understand and use Poisson as an approximation to the binomial distribution. Apply the result that the sum of independent Poisson random variables has a Poisson distribution. Use of the exponential distribution as a model for intervals between events.	Use of formula and tables/calculator for Poisson distribution. Knowledge and use of: If $\mathrm{X} \sim \mathrm{Po}(\lambda)$ then $\mathrm{E}(X) = \lambda$ and $\mathrm{Var}(X) = \lambda$ If $Y \sim \mathrm{Exp}(Y)$ then $\mathrm{E}(Y) = \frac{1}{\lambda}$ and $\mathrm{Var}(Y) = \frac{1}{\lambda^2}$ Learners will be expected to know that $\frac{\mathrm{d}}{\mathrm{d}x}\left(\mathrm{e}^{kx}\right) = k\mathrm{e}^{kx}$

<table>
<tr><th>Topics</th><th>Guidance</th></tr>
<tr><td>2.2.2 Exploring relationships between variables and goodness of fit of a model</td><td></td></tr>
<tr><td>Understand and use correlation and linear regression:

Explore the relationships between several variables.

Calculate and interpret
• Spearman’s rank correlation coefficient
• Pearson’s product-moment correlation coefficient.

Calculate and interpret the coefficients for a least squares regression line in context; interpolation and extrapolation.</td><td>To include tests for significance.
Excludes tied ranks.
Use of tables for Spearman's and Pearson's product moment correlation coefficients.
Be able to choose between Spearman’s rank correlation coefficient and Pearson’s product-moment correlation coefficient for a given context.

Including from summary statistics.</td></tr>
<tr><td>Understand and use the Chi-squared distribution:
Conduct goodness of fit test using $\sum \frac{(O-E)^2}{E}$, or equivalent form, as an approximate χ^2 statistic (for use with categorical data).

Use χ^2 test to test for association in a contingency table and interpret results</td><td>For use with binomial, discrete uniform and Poisson distributions, for known parameters only.

To include pooling.
Not including Yates continuity correction.</td></tr>
</table>

2.3 AS UNIT 3

Unit 3: Further Mechanics A
Written examination: 1 hour 30 minutes
$13\frac{1}{3}$% of A level qualification ($33\frac{1}{3}$% of AS qualification)
70 marks

The subject content is set out on the following pages. There is no hierarchy implied by the order in which the content is presented, nor should the length of the various sections be taken to imply any view of their relative importance.

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in AS Mathematics.
Where specific content requires knowledge of concepts or results from A2 Mathematics, this will be made explicit in the Guidance section of the content.

Topics	Guidance
2.3.1 Momentum and Impulse	
Understand and use momentum and impulse. Understand and use conservation of momentum. Understand and use Newton's Experimental Law for (i) the direct impact of two bodies moving in the same straight line, (ii) the impact of a body moving at right-angles to a plane.	Problems will be restricted to the one-dimensional case.
2.3.2 Hooke's Law, Work, Energy and Power	
Solve problems involving light strings and springs obeying Hooke's Law.	
Understand and use work, energy and power. Understand and use gravitational potential energy, kinetic energy, elastic energy. Understand and use conservation of energy. Understand and use the Work-energy Principle.	Calculation of work done by using change of energy.

Topics	Guidance
2.3.3 Circular Motion	
Understand and use circular motion.	Angular speed ω and the use of $v = r\omega$. Radial acceleration in circular motion in the form $r\omega^2$ and $\frac{v^2}{r}$.
Understand and use the motion of a particle in a horizontal circle with uniform angular speed.	Problems on banked tracks including the condition for no side slip. The conical pendulum. The motion of a particle in a horizontal circle where the particle is (i) constrained by two strings, (ii) threaded on one string, (iii) constrained by one string and a smooth horizontal surface.
Understand and use the motion in a vertical circle.	To include the determination of points where the circular motion breaks down (e.g. loss of contact with a surface or a string becoming slack). The condition for a particle to move in complete vertical circles when (i) it is attached to a light string, (ii) it is attached to a light rigid rod, (iii) it moves on the inside surface of a sphere. The tangential component of the acceleration is **not** required.
2.3.4 Differentiation and Integration of Vectors	
Differentiate and integrate vectors in component form with respect to a scalar variable. Understand and use vector quantities including displacement, velocity, acceleration, force and momentum.	Extends to vectors in 3 dimensions. Resultants of vector quantities. Simple applications including the relative motion of two objects and the determination of the shortest distance between them. *Knowledge of 3-D vectors is assumed.*

2.4 A2 UNIT 4

Unit 4: Further Pure Mathematics B
Written examination: 2 hours 30 minutes
35% of A level qualification
120 marks

This unit is **compulsory**.

The subject content is set out on the following pages. There is no hierarchy implied by the order in which the content is presented, nor should the length of the various sections be taken to imply any view of their relative importance.

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in A level Mathematics.

Topics	Guidance
2.4.1 Complex Numbers	
Understand de Moivre's theorem and use it to find multiple angle formulae and sums of series.	To include proof by induction of de Moivre's Theorem for positive integer values of n. For example, showing that $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$ and $\cos^4\theta = \frac{1}{8}(\cos 4\theta + 4\cos 2\theta + 3)$.
Know and use the definition $e^{i\theta} = \cos\theta + i\sin\theta$ and the form $z = re^{i\theta}$.	
Find the n distinct nth roots for $re^{i\theta}$ for $r \neq 0$ and know that they form the vertices of a regular n-gon in the Argand diagram.	
Use complex roots of unity to solve geometric problems.	

Topics	Guidance
2.4.2 Further Trigonometry	
Solve trigonometric equations. Use the formulae for $\sin A \pm \sin B$, $\cos A \pm \cos B$ and for $\sin x$, $\cos x$ and $\tan x$ in terms of t, where $t = \tan\frac{1}{2}x$. Find the general solution of trigonometric equations.	Questions aimed solely at proving identities will not be set. For example, $\cos\theta + \cos 2\theta + \cos 3\theta = 0$ and $2\sin x - \tan\frac{1}{2}x = 0$.
2.4.3 Matrices	
Calculate determinants up to 3 x 3 matrices and interpret as scale factors, including the effect on orientation.	
Calculate and use the inverse of non-singular 3 x 3 matrices.	Including knowledge of the term adjugate matrix.
Solve three linear simultaneous equations in three variables by use of the inverse matrix and by reduction to echelon form. Understand and use the determinantal condition for the solution of simultaneous equations which have a unique solution.	To include equations which (a) have a unique solution, (b) have non-unique solutions, (c) are not consistent.
Interpret geometrically the solution and failure of three simultaneous linear equations.	

Topics	Guidance
2.4.4 Further Algebra and Functions	
Find the Maclaurin series of a function (including the general term)	
Recognise and use the Maclaurin series for e^x, $\ln(1+x)$, $\sin x$, $\cos x$ and $(1+x)^n$, and be aware of the range of values of x for which they are valid.	Proof not required.
Understand and use partial fractions with denominators of the form $(ax+b)(cx^2+d)$.	
2.4.5 Further Calculus	
Evaluate improper integrals, where either the integrand is undefined at a value in the range of integration or the range of integration extends to infinity.	
Derive formulae for and calculate volumes of revolution.	Rotation may be about the x-axis or the y-axis.
Understand and evaluate the mean value of a function.	Mean value of a function $f(x) = \frac{1}{b-a}\int_a^b f(x)\mathrm{d}x$
Integrate using partial fractions (extend to quadratic factors (ax^2+c) in the denominator).	
Differentiate inverse trigonometric functions.	
Integrate functions of the form $\frac{1}{\sqrt{a^2-x^2}}$ and $\frac{1}{a^2+x^2}$ and be able to choose trigonometric substitutions to integrate associated functions.	

Topics	Guidance
2.4.6 Polar Coordinates	
Understand and use polar coordinates and be able to convert between polar and Cartesian coordinates.	Where $r \geq 0$ and the value of θ may be taken to be in either $[0, 2\pi)$ or $(-\pi, \pi]$.
Sketch curves with r given as a function of θ, including the use of trigonometric functions.	Candidates will be expected to sketch simple curves such as $r = a(b + c\cos\theta)$ and $r = a\cos n\theta$. Includes the location of points at which tangents are parallel to, or perpendicular to, the initial line.
Find the area enclosed by a polar curve.	Excludes the intersection of curves.
2.4.7 Hyperbolic functions	
Understand the definitions of hyperbolic functions, $\sinh x$, $\cosh x$ and $\tanh x$, including their domains and ranges, and be able to sketch their graphs.	$\cosh x = \frac{1}{2}\left(e^x + e^{-x}\right)$, $\sinh x = \frac{1}{2}\left(e^x - e^{-x}\right)$ $\tanh x = \frac{\sinh x}{\cosh x} = \frac{\left(e^x - e^{-x}\right)}{\left(e^x + e^{-x}\right)}$ Know and use the formulae for $\sinh(A \pm B)$, $\cosh(A \pm B)$, $\tanh(A \pm B)$, $\sinh 2A$, $\cosh 2A$ and $\tanh 2A$. Knowledge and use of the identity $\cosh^2 A - \sinh^2 A \equiv 1$ and its equivalents.
Differentiate and integrate hyperbolic functions.	eg. Differentiate $\sinh 2x$, $x\cosh^2 x$
Understand and be able to use the definitions of the inverse hyperbolic functions and their domains and ranges.	$\sinh^{-1} x = \ln\left[x + \sqrt{x^2 + 1}\right]$ $\cosh^{-1} x = \ln\left[x + \sqrt{x^2 - 1}\right]$, $x \geq 1$ $\tanh^{-1} x = \frac{1}{2}\ln\left[\frac{1+x}{1-x}\right]$, $-1 < x < 1$

Topics	Guidance
Derive and use the logarithmic forms of the inverse hyperbolic functions.	
Integrate functions of the form $\frac{1}{\sqrt{x^2+a^2}}$ and $\frac{1}{\sqrt{x^2-a^2}}$, and be able to choose substitutions to integrate associated functions.	
2.4.8 Differential equations	
Find and use an integrating factor to solve differential equations of the form $\frac{\mathrm{d}y}{\mathrm{d}x}+P(x)y=Q(x)$ and recognise when it is appropriate to do so.	
Find both general and particular solutions to differential equations.	
Use differential equations in modelling in a variety of contexts.	Contexts will **not** include mechanics contexts.
Solve differential equations of the form $y''+ay'+by=0$, where a and b are constants, by using the auxiliary equation.	
Solve differential equations of the form $y''+ay'+by=f(x)$, where a and b are constants, by solving the homogenous case and adding a particular integral to the complementary function (in cases where $f(x)$ is a polynomial, exponential or trigonometric function).	$f(x)$ will have one of the forms $A+Bx$, cx^2+dx+e, $k\mathrm{e}^{qx}$ or $m\cos\omega x+n\sin\omega x$.
Understand and use the relationship between the cases when the discriminant of the auxiliary equation is positive, zero and negative and the form of solution of the differential equation.	

Topics	Guidance
Analyse and interpret models of situations with one independent variable and two dependent variables as a pair of coupled 1st order simultaneous equations and be able to solve them.	For example, predator-prey models. Restricted to first order differential equations of the form $\frac{\mathrm{d}x}{\mathrm{d}t} = ax + by + f(t)$ $\frac{\mathrm{d}y}{\mathrm{d}t} = cx + dy + g(t)$

2.5 A2 UNIT 5

Unit 5: Further Statistics B
Written examination: 1 hour 45 minutes
25% of A level qualification
80 marks

Candidates will choose **either** Unit 5 **or** Unit 6.

The subject content is set out on the following pages. There is no hierarchy implied by the order in which the content is presented, nor should the length of the various sections be taken to imply any view of their relative importance.

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in A Level Mathematics.

Topics	Guidance
2.5.1 Samples and Populations	
Understand and use unbiased estimators: Understand and use the variance criterion for choosing between unbiased estimators. Understand and use unbiased estimators of a probability and of a population mean and their standard errors. Understand and use an unbiased estimator of a population variance.	Use of $s^2 = \frac{1}{n-1}\sum\left(x_i - \bar{x}\right)^2$.

Topics	Guidance
2.5.2 Statistical Distributions	
Understand and use the result that a linear combination of independent normally distributed random variables has a normal distribution. Understand and use the fact that the distribution of the mean of a random sample from a normal distribution with known mean and variance is also normal. Know and use the Central Limit Theorem: Understand and use the fact that the distribution of the mean of a large random sample from any distribution with known mean and variance is approximately normally distributed.	For a population with mean μ and variance σ^2, for large n $\bar{X} \approx \ \sim N\left(\mu, \frac{\sigma^2}{n}\right)$
2.5.3 Hypothesis Testing	
Understand and use tests for: (a) a specified mean of any distribution whose variance is estimated from a large sample. (b) difference of two means for two independent normal distributions with known variances. (c) a specified mean of a normal distribution with unknown variance. Interpret results for these tests in context.	Using the Central Limit Theorem. The specified difference may be different from zero. To include estimating the variance from the data and using the Student's *t*-distribution. The significance level will be given and questions involving the Student's *t*-distribution will not require the calculation of *p*-values.
Non-parametric tests: Understand and use Mann-Whitney and Wilcoxon signed-rank tests, understanding appropriate test selection and interpreting the results in context.	Alternative tests for when a distributional model cannot be assumed. Excludes tied ranks.

Topics	Guidance
2.5.4 Estimation	
Understand and use confidence intervals: Understand and use confidence limits for (a) the mean of a normal distribution with (i) known variance and (ii) unknown variance, (b) the difference between the means of two normal distributions whose variances are known. Understand and use approximate confidence limits, given large samples, for a probability or a proportion. Interpret results in practical contexts.	Candidates will be expected to be familiar with the term 'confidence interval', including its interpretation. Estimating the variance from the data and using the Student's *t*-distribution. Using a normal approximation.

2.6 A2 UNIT 6

Unit 6: Further Mechanics B
Written examination: 1 hour 45 minutes
25% of A level qualification
80 marks

Candidates will choose **either** Unit 5 **or** Unit 6.

The subject content is set out on the following pages. There is no hierarchy implied by the order in which the content is presented, nor should the length of the various sections be taken to imply any view of their relative importance.

Candidates will be expected to be familiar with the knowledge, skills and understanding implicit in A Level Mathematics.

Topics	Guidance
2.6.1 Rectilinear motion	
Form and solve simple equations of motion in which (i) acceleration is given as a function of time, displacement or velocity, (ii) velocity is given as a function of time or displacement.	To include use of $\frac{\mathrm{d}^2x}{\mathrm{d}t^2}=\frac{\mathrm{d}v}{\mathrm{d}t}=v\frac{\mathrm{d}v}{\mathrm{d}x}$.
2.6.2 Momentum and Impulse	
Understand and use momentum and impulse in two dimensions, using vectors.	

Topics	Guidance
2.6.3 Moments and Centre of Mass	
Understand and use the centre of mass of a coplanar system of particles. Understand and use the centre of mass of uniform laminae: triangles, rectangles, circles, semicircles, quarter-circles and composite shapes. Solve problems involving simple cases of equilibrium of a plane lamina and/or a coplanar system of particles connected by light rods.	Candidates will be expected to be familiar with the term 'centre of gravity'. The lamina or system of particles may be suspended from a fixed point.
Understand and use the centre of mass of uniform rigid bodies and composite bodies.	The use of symmetry and/or integration to determine the centre of mass of a uniform body.
2.6.4 Equilibrium of Rigid Bodies	
Understand and use the equilibrium of a single rigid body under the action of coplanar forces where the forces are not all parallel.	Problems may include rods resting against rough or smooth walls and on rough ground. Consideration of jointed rods is not required. Questions involving toppling will not be set.
2.6.5 Differential Equations	
Use differential equations in modelling in kinematics.	To include the use of first and second order differential equations.
Understand and use simple harmonic motion.	Candidates will be expected to set up the differential equation of motion, identify the period, amplitude and appropriate forms of solution. Candidates may quote formulae in problems unless the question specifically requires otherwise. Questions may involve light elastic strings or springs. Questions may require the refinement of the mathematical model to include damping. Angular S.H.M. is not included.

3 ASSESSMENT

3.1 Assessment objectives and weightings

Below are the assessment objectives for this specification. Learners must:

AO1
Use and apply standard techniques
Learners should be able to:
- select and correctly carry out routine procedures; and
- accurately recall facts, terminology and definitions

AO2
Reason, interpret and communicate mathematically
Learners should be able to:
- construct rigorous mathematical arguments (including proofs);
- make deductions and inference;
- assess the validity of mathematical arguments;
- explain their reasoning; and
- use mathematical language and notation correctly.

AO3
Solve problems within mathematics and in other contexts
Learners should be able to:
- translate problems in mathematical and non-mathematical contexts into mathematical processes;
- interpret solutions to problems in their original context, and, where appropriate, evaluate their accuracy and limitations;
- translate situations in context into mathematical models;
- use mathematical models; and
- evaluate the outcomes of modelling in context, recognise the limitations of models and, where appropriate, explain how to refine them.

Approximate assessment objective weightings are shown below as a percentage of the full A level, with AS weightings in brackets.

	AO1	AO2	AO3	Total
AS Unit 1	8% (20%)	$2\frac{2}{3}$% ($6\frac{2}{3}$%)	$2\frac{2}{3}$% ($6\frac{2}{3}$%)	$13\frac{1}{3}$% ($33\frac{1}{3}$%)
AS Unit 2	6% (15%)	$3\frac{2}{3}$% ($9\frac{1}{6}$%)	$3\frac{2}{3}$% ($9\frac{1}{6}$%)	$13\frac{1}{3}$% ($33\frac{1}{3}$%)
AS Unit 3	6% (15%)	$3\frac{2}{3}$% ($9\frac{1}{6}$%)	$3\frac{2}{3}$% ($9\frac{1}{6}$%)	$13\frac{1}{3}$% ($33\frac{1}{3}$%)
Total for AS units only	20%	10%	10%	40%
A2 Unit 4	20%	$7\frac{1}{2}$%	$7\frac{1}{2}$%	35%
A2 Unit 5 (option)	10%	$7\frac{1}{2}$%	$7\frac{1}{2}$%	25%
A2 Unit 6 (option)				
Total for A2 units only	30%	15%	15%	60%
Final Total A Level	50%	25%	25%	100%

Use of technology

The use of technology, in particular mathematical and statistical graphing tools and spreadsheets, permeates the study of GCE AS and A Level Further Mathematics.

A calculator is required for use in all assessments in this specification.

Calculators used must include the following features:

- an iterative function;
- the ability to compute summary statistics and access probabilities from standard statistical distributions.

Calculators must also meet the regulations set out below.

Calculators must be:	**Calculators must not:**

• of a size suitable for use on the desk;
• either battery or solar powered;
• free of lids, cases and covers which have printed instructions or formulas.
 	
The candidate is responsible for the following:
• the calculator's power supply;
• the calculator's working condition;
• clearing anything stored in the calculator.
	• be designed or adapted to offer any of these facilities: -
	  • language translators;
	  • symbolic algebra manipulation;
	  • symbolic differentiation or integration;
	  • communication with other machines or the internet;
	• be borrowed from another candidate during an examination for any reason;*
	• have retrievable information stored in them - this includes:
	  • databanks;
	  • dictionaries;
	  • mathematical formulas;
	  • text.

* An invigilator may give a candidate a replacement calculator.

Formula Booklet

A formula booklet will be required in all examinations. This will exclude any formulae listed in Appendix B. Copies of the formula booklet may be obtained from the WJEC.

Statistical Tables

Candidates may use a book of statistical tables for Unit 2 and Unit 5.

The following book of statistical tables is allowed in the examinations:

- Elementary Statistical Tables (RND/WJEC Publications).

4 TECHNICAL INFORMATION

4.1 Making entries

This is a unitised specification which allows for an element of staged assessment.

Assessment opportunities will be available in the summer assessment period each year, until the end of the life of the specification.

Unit 1, Unit 2 and Unit 3 will be available in 2018 (and each year thereafter) and the AS qualification will be awarded for the first time in summer 2018.

Unit 4, Unit 5 and Unit 6 will be available in 2019 (and each year thereafter) and the A level qualification will be awarded for the first time in summer 2019.

Candidates may re-sit units **ONCE ONLY** prior to certification for the qualification, with the better result contributing to the qualification. Individual unit results, prior to the certification of the qualification, have a shelf-life limited only by that of the qualification.

A candidate may retake the whole qualification more than once.

The entry codes appear below.

	Title	**Entry codes**	
		English-medium	**Welsh-medium**
AS Unit 1	Further Pure Mathematics A	2305U1	2305N1
AS Unit 2	Further Statistics A	2305U2	2305N2
AS Unit 3	Further Mechanics A	2305U3	2305N3
A2 Unit 4	Further Pure Mathematics B	1305U4	1305N4
A2 Unit 5	Further Statistics B	1305U5	1305N5
A2 Unit 6	Further Mechanics B	1305U6	1305N6
AS Qualification cash-in		2305QS	2305CS
A level Qualification cash-in		1305QS	1305CS

The current edition of our *Entry Procedures and Coding Information* gives up-to-date entry procedures.

There is no restriction on entry for this specification with any other WJEC AS or A level specification.

4.2 Grading, awarding and reporting

The overall grades for the GCE AS qualification will be recorded as a grade on a scale A to E. The overall grades for the GCE A level qualification will be recorded as a grade on a scale A* to E. Results not attaining the minimum standard for the award will be reported as U (unclassified). Unit grades will be reported as a lower case letter a to e on results slips but not on certificates.

The Uniform Mark Scale (UMS) is used in unitised specifications as a device for reporting, recording and aggregating candidates' unit assessment outcomes. The UMS is used so that candidates who achieve the same standard will have the same uniform mark, irrespective of when the unit was taken. Individual unit results and the overall subject award will be expressed as a uniform mark on a scale common to all GCE qualifications. An AS GCE has a total of 240 uniform marks and an A level GCE has a total of 600 uniform marks. The maximum uniform mark for any unit depends on that unit's weighting in the specification.

Uniform marks correspond to unit grades as follows:

		Unit grade				
Unit weightings	**Maximum unit uniform mark**	**a**	**b**	**c**	**d**	**e**
Unit 1 ($13\frac{1}{3}$%)	80 (raw mark max=70)	64	56	48	40	32
Unit 2 ($13\frac{1}{3}$%)	80 (raw mark max=70)	64	56	48	40	32
Unit 3 ($13\frac{1}{3}$%)	80 (raw mark max=70)	64	56	48	40	32
Unit 4 (35%)	210 (raw mark max=120)	168	147	126	105	84
Unit 5 (25%) **Unit 6 (25%)**	150 (raw mark max=80)	120	105	90	75	60

The uniform marks obtained for each unit are added up and the subject grade is based on this total.

		Qualification grade				
	Maximum uniform marks	**A**	**B**	**C**	**D**	**E**
GCE AS	240	192	168	144	120	96
GCE A level	600	480	420	360	300	240

At A level, Grade A* will be awarded to candidates who have achieved a Grade A (480 uniform marks) in the overall A level qualification and at least 90% of the total uniform marks for the A2 units (324 uniform marks).

APPENDIX A

Mathematical Notation

The tables below set out the notation that must be used in the WJEC GCE AS and A Level Further Mathematics specification. Learners will be expected to understand this notation without the need for further explanation.

AS learners will be expected to understand notation that relates to AS content, and will not be expected to understand notation that relates only to A Level content.

1	Set Notation	
1.1	$\in$	is an element of
1.2	$\notin$	is not an element of
1.3	$\subseteq$	is a subset of
1.4	$\subset$	is a proper subset of
1.5	$\{x_1, x_2, \ldots\}$	the set with elements $x_1, x_2, \ldots$
1.6	$\{x: \ldots\}$	the set of all x such that …
1.7	$\mathrm{n}(A)$	the number of elements in set A
1.8	$\emptyset$	the empty set
1.9	$\boldsymbol{\varepsilon}$	the universal set
1.10	A'	the complement of the set A
1.11	$\mathbb{N}$	the set of natural numbers, $\{1, 2, 3, \ldots\}$
1.12	$\mathbb{Z}$	the set of integers, $\{0, \pm 1, \pm 2, \pm 3, \ldots\}$
1.13	$\mathbb{Z}^+$	the set of positive integers, $\{1, 2, 3, \ldots\}$
1.14	$\mathbb{Z}_0^+$	the set of non-negative integers, $\{0, 1, 2, 3, \ldots\}$
1.15	$\mathbb{R}$	the set of real numbers
1.16	$\mathbb{Q}$	the set of rational numbers $\left\{\frac{p}{q} : p \in \mathbb{Z}, q \in \mathbb{Z}^+\right\}$
1.17	$\cup$	union
1.18	$\cap$	intersection
1.19	(x, y)	the ordered pair x, y
1.20	$[a, b]$	the closed interval $\{x \in \mathbb{R}: a \leq x \leq b\}$
1.21	$[a, b)$	the interval $\{x \in \mathbb{R}: a \leq x < b\}$
1.22	$(a, b]$	the interval $\{x \in \mathbb{R}: a < x \leq b\}$
1.23	(a, b)	the open interval $\{x \in \mathbb{R}: a < x < b\}$
1.24	$\mathbb{C}$	the set of complex numbers

2	**Miscellaneous Symbols**	
2.1	$=$	is equal to
2.2	$\neq$	is not equal to
2.3	$\equiv$	is identical to or congruent to
2.4	$\approx$	is approximately equal to
2.5	∞	infinity
2.6	$\propto$	is proportional to
2.7	$\therefore$	therefore
2.8	$\because$	because
2.9	$<$	is less than
2.10	$\leqslant, \leq$	is less than or equal to, is not greater than
2.11	$>$	is greater than
2.12	$\geqslant, \geq$	is greater than or equal to, is not less than
2.13	$p \Rightarrow q$	p implies q (if p then q)
2.14	$p \Leftarrow q$	p is implied by q (if q then p)
2.15	$p \Leftrightarrow q$	p implies and is implied by q (p is equivalent to q)
2.16	a	first term for an arithmetic or geometric sequence
2.17	l	last term for an arithmetic sequence
2.18	d	common difference for an arithmetic sequence
2.19	r	common ratio for a geometric sequence
2.20	S_n	sum to n terms of a sequence
2.21	S_∞	sum to infinity of a sequence
3	**Operations**	
3.1	$a + b$	a plus b
3.2	$a - b$	a minus b
3.3	$a \times b, a\,b, a.b$	a multiplied by b
3.4	$a \div b, \frac{a}{b}$	a divided by b
3.5	$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \ldots + a_n$
3.6	$\prod_{i=1}^{n} a_i$	$a_1 \times, a_2 \times, \ldots \times\ a_n$
3.7	$\sqrt{a}$	the non-negative square root of a
3.8	$\lvert a \rvert$	the modulus of a
3.9	$n!$	n factorial: $n! = n \times (n-1) \times \ldots \times 2 \times 1$, $n \in \mathbb{N}; 0! = 1$

3.10	$\binom{n}{r}$, ${}^nC_r$, ${}_nC_r$	the binomial coefficient $\frac{n!}{r!(n-r)!}$ for $n, r \in \mathbb{Z}_0^+$, $r \leqslant n$ or $\frac{n(n-1)\dots(n-r+1)}{r!}$ for $n \in \mathbb{Q}, r \in \mathbb{Z}_0^+$
4	**Functions**	
4.1	$\mathrm{f}(x)$	the value of the function f at x
4.2	$\mathrm{f} : x \mapsto y$	the function f maps the element x to the element y
4.3	f^{-1}	the inverse function of the function f
4.4	gf	the composite function of f and g which is defined by $\mathrm{gf}(x) = \mathrm{g}(\mathrm{f}(x))$
4.5	$\lim_{x \to a} \mathrm{f}(x)$	the limit of $\mathrm{f}(x)$ as x tends to a
4.6	$\Delta x, \delta x$	an increment of x
4.7	$\frac{\mathrm{d}y}{\mathrm{d}x}$	the derivative of y with respect to x
4.8	$\frac{\mathrm{d}^n y}{\mathrm{d}x^n}$	the n^{th} derivative of y with respect to x
4.9	$\mathrm{f}'(x), \mathrm{f}''(x), \dots, \mathrm{f}^n\,(x)$	the first, second …, n^{th} derivatives of $\mathrm{f}(x)$ with respect to x
4.10	$\dot{x}, \ddot{x}, \dots$	the first, second, … derivatives of x with respect to t
4.11	$\int y\ \mathrm{d}x$	the indefinite integral of y with respect to x
4.12	$\int_a^b y\ \mathrm{d}x$	the definite integral of y with respect to x between the limits $x = a$ and $x = b$
5	**Exponential and Logarithmic Functions**	
5.1	e	base of natural logarithms
5.2	e^x, $\exp x$	exponential function of x
5.3	$\log_a x$	logarithm to the base a of x
5.4	$\ln x$, $\log_{\mathrm{e}} x$	natural logarithm of x
6	**Trigonometric Functions**	
6.1	$\left.\begin{matrix}\sin, \cos, \tan,\\ \mathrm{cosec}, \sec, \cot\end{matrix}\right\}$	the trigonometric functions
6.2	$\left.\begin{matrix}\sin^{-1}, \cos^{-1}, \tan^{-1}\\ \arcsin, \arccos, \arctan\end{matrix}\right\}$	the inverse trigonometric functions
6.3	$\circ$	degrees
6.4	rad	radians
6.5	$\left.\begin{matrix}\mathrm{cosec}^{-1}, \sec^{-1}, \cot^{-1}\\ \mathrm{arccosec}, \mathrm{arcsec}, \mathrm{arccot}\end{matrix}\right\}$	the inverse trigonometric functions

6.6	$\left.\begin{matrix}\sinh, \cosh, \tanh, \\ \operatorname{cosech}, \operatorname{sech}, \coth\end{matrix}\right\}$	the hyperbolic functions
6.7	$\left.\begin{matrix}\sinh^{-1}, \cosh^{-1}, \tanh^{-1}, \\ \operatorname{cosech}^{-1}, \operatorname{sech}^{-1}, \coth^{-1}\end{matrix}\right\}$ $\left.\begin{matrix}\operatorname{arsinh}, \operatorname{arcosh}, \operatorname{artanh}, \\ \operatorname{arcosech}, \operatorname{arsech}, \operatorname{arcoth}\end{matrix}\right\}$	the inverse hyperbolic functions
7	**Complex Numbers**	
7.1	i , j	square root of -1
7.2	$x + \mathrm{i}y$	complex number with real part x and imaginary part y
7.3	$\mathrm{r}(\cos\theta + \mathrm{i}\sin\theta)$	modulus argument form of a complex number with modulus r and argument θ
7.4	z	a complex number, $z = x + \mathrm{i}y = r(\cos\theta + \mathrm{i}\sin\theta)$
7.5	$\mathrm{Re}(z)$	the real part of z, Re $(z) = x$
7.6	$\mathrm{Im}(z)$	the imaginary part of z, $\mathrm{Im}(z)=y$
7.7	$\lvert z\rvert$	the modulus of z, $\lvert z\rvert = \sqrt{x^2+y^2}$
7.8	$\arg(z)$	the argument of z, $\arg(z) = \theta$, $-\pi < \theta \leq \pi$
7.9	z^* or $\bar{z}$	the complex conjugate of z, $x - \mathrm{i}y$
8	**Matrices**	
8.1	$\mathbf{M}$	a matrix $\mathbf{M}$
8.2	$\mathbf{0}$	zero matrix
8.3	$\mathbf{I}$	identity matrix
8.4	$\mathbf{M}^{-1}$	the inverse of the matrix $\mathbf{M}$
8.5	$\mathbf{M}^{\mathrm{T}}$	the transpose of the matrix $\mathbf{M}$
8.6	Δ, $\det \mathbf{M}$ or $\lvert\mathbf{M}\rvert$	the determinant of the square matrix $\mathbf{M}$
8.7	**Mr**	image of column vector $\mathbf{r}$ under the transformation associated with the matrix $\mathbf{M}$
9	**Vectors**	
9.1	$\mathbf{a}$, $\underline{a}$, $\underset{\sim}{a}$	the vector $\mathbf{a}$, $\underline{a}$, $\underset{\sim}{a}$; these alternatives apply throughout section 9
9.2	$\overrightarrow{\mathrm{AB}}$	the vector represented in magnitude and direction by the directed line segment AB
9.3	$\hat{\mathbf{a}}$	a unit vector in the direction of $\mathbf{a}$
9.4	$\mathbf{i}, \mathbf{j}, \mathbf{k}$	unit vectors in the directions of the cartesian coordinate axes
9.5	$\lvert\mathbf{a}\rvert$, a	the magnitude of $\mathbf{a}$

9.6	$\lvert\overrightarrow{\mathrm{AB}}\rvert$, AB	the magnitude of $\overrightarrow{\mathrm{AB}}$
9.7	$\begin{pmatrix} a \\ b \end{pmatrix}, a\mathbf{i} + b\mathbf{j}$	column vector and corresponding unit vector notation
9.8	$\mathbf{r}$	position vector
9.9	$\mathbf{s}$	displacement vector
9.10	$\mathbf{v}$	velocity vector
9.11	$\mathbf{a}$	acceleration vector
9.12	$\mathbf{a.b}$	the scalar product of $\mathbf{a}$ and $\mathbf{b}$
10	**Differential Equations**	
10.1	ω	angular speed
11	**Probability and Statistics**	
11.1	A, B, C, etc.	events
11.2	$A \cup B$	union of the events A and B
11.3	$A \cap B$	intersection of the events A and B
11.4	$\mathrm{P}(A)$	probability of the event A
11.5	A'	complement of the event A
11.6	$\mathrm{P}(A\|B)$	probability of the event A conditional on the event B
11.7	X, Y, R, etc.	random variables
11.8	x, y, r, etc.	values of the random variables X, Y, R etc
11.9	$x_1, x_2, \ldots$	values of observations
11.10	$f_1, f_2, \ldots$	frequencies with which the observations $x_1, x_2, \ldots$ occur
11.11	$\mathrm{p}(x)$, $\mathrm{P}(X{=}x)$	probability function of the discrete random variable X
11.12	$p_1, p_2, \ldots$	probabilities of the values $x_1, x_2, \ldots$ of the discrete random variable X
11.13	$\mathrm{E}(X)$	expectation of the random variable X
11.14	$\mathrm{Var}(X)$	variance of the random variable X
11.15	$\sim$	has the distribution
11.16	$\mathrm{B}(n, p)$	binomial distribution with parameters n and p, where n is the number of trials and p is the probability of success in a trial
11.17	q	$q = 1 - p$ for binomial distribution
11.18	$\mathrm{N}(\mu, \sigma^2)$	Normal distribution with mean μ and variance σ^2
11.19	$Z \sim \mathrm{N}(0,1)$	standard Normal distribution
11.20	ϕ	probability density function of the standardised Normal variable with distribution $\mathrm{N}(0,1)$

11.21	Φ	corresponding cumulative distribution function
11.22	μ	population mean
11.23	σ^2	population variance
11.24	σ	population standard deviation
11.25	$\bar{x}$	sample mean
11.26	s^2	sample variance
11.27	s	sample standard deviation
11.28	H_0	null hypothesis
11.29	H_1	alternative hypothesis
11.30	r	product moment correlation coefficient for a sample
11.31	ρ	product moment correlation coefficient for a population
12	**Mechanics**	
12.1	kg	kilograms
12.2	m	metres
12.3	km	kilometres
12.4	$\mathrm{m/s},\ \mathrm{m\ s^{-1}}$	metres per second (velocity)
12.5	$\mathrm{m/s^2},\ \mathrm{m\ s^{-2}}$	metres per second per second (acceleration)
12.6	F	force or resultant force
12.7	N	newton
12.8	$\mathrm{N\ m}$	newton metre (moment of a force)
12.9	t	time
12.10	s	displacement
12.11	u	initial velocity
12.12	v	velocity or final velocity
12.13	a	acceleration
12.14	g	acceleration due to gravity
12.15	μ	coefficient or friction

APPENDIX B

Mathematical Formulae and Identities

Learners must be able to use the following formulae and identities for GCE AS and A Level Further Mathematics, without these formulae and identities being provided, either in these forms or in equivalent forms. These formulae and identities may only be provided where they are the starting point for a proof or as a result to be proved.

Pure Mathematics

Quadratic Equations

$ax^2 + bx + c = 0$ has roots $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Laws of Indices

$a^x a^y \equiv a^{x+y}$

$a^x \div a^y \equiv a^{x-y}$

$(a^x)^y \equiv a^{xy}$

Laws of Logarithms

$x = a^n \Leftrightarrow n = \log_a x$ for $a > 0$ and $x > 0$

$\log_a x + \log_a y \equiv \log_a (xy)$

$\log_a x - \log_a y \equiv \log_a \left(\dfrac{x}{y}\right)$

$k \log_a x \equiv \log_a \left(x^k\right)$

Coordinate Geometry

A straight line graph, gradient m passing through (x_1, y_1) has equation

$y - y_1 = m(x - x_1)$

Straight lines with gradients m_1 and m_2 are perpendicular when $m_1 m_2 = -1$

Sequences

General term of arithmetic progression:

$u_n = a + (n-1)d$

General term of a geometric progression:

$u_n = ar^{n-1}$

Trigonometry

In the triangle ABC

Sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Cosine rule: $a^2 = b^2 + c^2 - 2b\cos A$

Area = $\dfrac{1}{2}ab\sin C$

$\cos^2 A + \sin^2 A \equiv 1$
$\sec^2 A \equiv 1 + \tan^2 A$
$\operatorname{cosec}^2 A \equiv 1 + \cot^2 A$
$\sin 2A \equiv 2\sin A\cos A$
$\cos 2A \equiv \cos^2 A - \sin^2 A$

$\tan 2A \equiv \dfrac{2\tan A}{1 - \tan^2 A}$

Mensuration

Circumference and Area of circle, radius r and diameter d:
$C = 2\pi r = \pi d \quad A = \pi r^2$

Pythagoras' Theorem: In any right-angled triangle where a, b and c are the lengths of the sides and c is the hypotenuse:
$c^2 = a^2 + b^2$

Area of trapezium = $\dfrac{1}{2}(a+b)h,$ where a and b are the lengths of the parallel sides and h is their perpendicular separation.

Volume of a prism = area of cross section × length

For a circle of radius, r, where an angle at the centre of θ radians subtends an arc of length s and encloses an associated sector of area A:
$s = r\theta \quad A = \dfrac{1}{2}r^2\theta$

Complex Numbers

For two complex numbers $z_1 = r_1 e^{i\theta_1}$ and $z_2 = r_2 e^{i\theta_2}$:

$z_1 z_2 = r_1 r_2 e^{i(\theta_1 + \theta_2)}$

$\dfrac{z_1}{z_2} = \dfrac{r_1}{r_2} e^{i(\theta_1 - \theta_2)}$

Loci in the Argand diagram:

$\lvert z-a\rvert = r$	is a circle radius r centred at a
arg $(z-a)=\theta$	is a half line drawn from a at an angle θ to a line parallel to the positive real axis

Exponential Form:
$e^{i\theta} = \cos\theta + i\sin\theta$

Matrices

For a 2 by 2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ the determinant $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

The inverse is $\frac{1}{\Delta}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

The transformation represented by matrix $\mathbf{AB}$ is the transformation represented by matrix $\mathbf{B}$ followed by the transformation represented by matrix $\mathbf{A}$.
For matrices $\mathbf{A}, \mathbf{B}$:

$$(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$$

Algebra

$$\sum_{r=1}^{n} r = \frac{1}{2}n(n+1)$$

For $ax^2 + bx + c = 0$ with roots α and β:

$$\alpha + \beta = \frac{-b}{a}, \quad \alpha\beta = \frac{c}{a}$$

For $ax^3 + bx^2 + cx + d = 0$ with roots α, β and γ:

$$\sum\alpha = \frac{-b}{a}, \quad \sum\alpha\beta = \frac{c}{a}, \quad \sum\alpha\beta\gamma = \frac{-d}{a}$$

Hyperbolic Functions

$$\cosh x \equiv \frac{1}{2}\left(e^x + e^{-x}\right)$$

$$\sinh x \equiv \frac{1}{2}\left(e^x - e^{-x}\right)$$

$$\tanh x \equiv \frac{\sinh x}{\cosh x}$$

Calculus and Differential Equations

Differentiation

Function	Derivative
x^n	nx^{n-1}
$\sin kx$	$k\cos kx$
$\cos kx$	$-k\sin kx$
$\sinh kx$	$k\cosh kx$
$\cosh kx$	$k\sinh kx$
e^{kx}	ke^{kx}
$\ln x$	$\frac{1}{x}$
$f(x)+g(x)$	$f'(x)+g'(x)$
$f(x)g(x)$	$f'(x)g(x)+f(x)g'(x)$
$f(g(x))$	$f'(g(x))g'(x)$

Integration

Function	Integral
x^n	$\frac{1}{n+1}x^{n+1}+c, n \neq -1$
$\cos kx$	$\frac{1}{k}\sin kx+c$
$\sin kx$	$-\frac{1}{k}\cos kx+c$
$\cosh kx$	$\frac{1}{k}\sinh kx+c$
$\sinh kx$	$\frac{1}{k}\cosh kx+c$
e^{kx}	$\frac{1}{k}e^{kx}+c$
$\frac{1}{x}$	$\ln\lvert x\rvert+c, x \neq 0$
$f'(x)+g'(x)$	$f(x)+g(x)+c$
$f'(g(x))g'(x)$	$f(g(x))+c$

Area under a curve $= \int_a^b y\,dx \qquad (y \geq 0)$

Volumes of revolution about x and y axes:

$$V_x = \pi \int_a^b y^2 dx$$

$$V_y = \pi \int_c^d x^2 dy$$

Simple Harmonic Motion:

$\ddot{x} = w^2 x$

Vectors

$$|\ x\mathbf{i} + y\mathbf{j} + z\mathbf{k}\ | = \sqrt{\left(x^2 + y^2 + z^2\right)}$$

Scalar product of two vectors $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a^3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ is

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} . \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3 = |\mathbf{a}||\mathbf{b}|\cos\theta$$

Where θ is the acute angle between the vector **a** and **b**

The equation of the line through the point with position vector **a** parallel to vector **b** is:

$\mathbf{r} = \mathbf{a} + t\mathbf{b}$

The equation of the plane containing the point with position vector **a** and perpendicular to vector **n** is:

$(\mathbf{r} - \mathbf{a}).\mathbf{n} = 0$

Mechanics

Forces and Equilibrium

Weight = mass $\times g$

Friction $F \leq \mu R$

Newton's second law in the form: $F = ma$

Kinematics

For motion in a straight line with variable acceleration:

$$v = \frac{dr}{dt} \qquad a = \frac{dv}{dt} = \frac{d^2 r}{dt^2}$$

$$r = \int v\,dt \qquad v = \int a\,dt$$

Statistics

The mean of a set of data $\bar{x} = \frac{\sum x}{n} = \frac{\sum fx}{\sum f}$

The standard Normal variable: $Z = \frac{x - \mu}{\sigma}$ where $X \sim N\left(\mu, \sigma^2\right)$